Third Edition

Fundamentals of BUSINESS MATHEMATICS in Canada

F. Ernest Jerome Jackie Shemko

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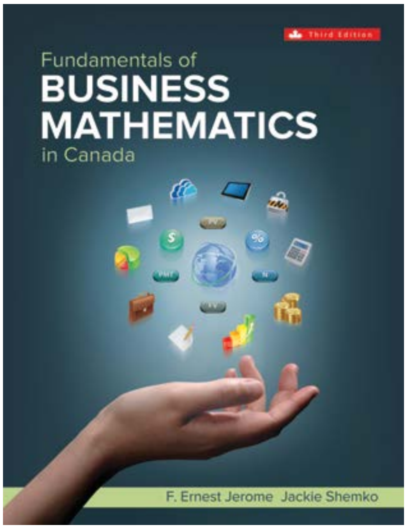

Business mathematics continues to be an important foundational course in most business administration programs in Canadian colleges. The business mathematics curriculum helps students build the mathematics skills they will need as they progress through their studies in accounting, marketing, operations management, human resources management, and business information systems.

Fundamentals of Business Mathematics in Canada, Third Edition is a comprehensive yet accessible text designed primarily for one-semester business mathematics courses. In recognition of the time pressures inherent in single-semester mathematics courses, the content and chapter features have been selected to support the development of essential learning outcomes. The focus is on clear exposition, with careful consideration given to the topic areas for which, based on the authors' and reviewers' experience, students need additional support. The text presents a carefully selected suite of categorized problems—basic, intermediate, and advanced—allowing faculty to easily tailor in-class examples and homework assignments by level of difficulty. All topics typically addressed in business mathematics courses are presented. The structure of the text allows faculty to customize their delivery as they decide how far into each topic they wish to take their students.

Jerome/Shemko is suitable for courses that emphasize either an algebraic approach or a preprogrammed financial calculator approach to compound interest problems. Both algebraic solutions and financial calculator solutions are presented in most Example problems for compound-interest topics.

NEW TO THIS EDITION

This edition incorporates suggestions from users and reviewers of the previous editions. The aim is to continue to present an accessible and flexible resource that encourages students to build skills and apply those skills with confidence to real-world problems.

- **Updated problems:** Although low interest rates persist in Canada, it is important that students appreciate how individuals and businesses are affected when rates climb. While the majority of Exercise and Example problems use rates that are in line with our current low interest rate environment, we have provided many examples with slightly higher interest rates, to help reinforce the idea that businesses, consumers and investors should appreciate the impact that higher rates could have on their own finances and the economy as a whole.
- **New Math Apps features:** Six new Math Apps boxes have been written to help illustrate the "real-world" application of chapter concepts. "Striking the Balance with Provincial Sales Taxes" in Chapter 1 compares the provincial sales tax frameworks in British Columbia, Alberta, and Nova Scotia, showing how provincial governments have struggled to balance consumer preferences with their own fiscal challenges. In Chapter 4, "The Cost of a 'Cash-less' Society" provides an update about a Competition Bureau investigation into credit card merchant fees, and how that prompted Visa and MasterCard to commit to a voluntary schedule of fee reductions. In Chapter 7, "What to Do with a Small Amount of Savings in a Low Interest Rate Environment?" helps reinforce the idea that while low interest rates are a boon for borrowers, they pose significant challenges for investors, especially those with only small amounts to work with. Also in Chapter 7, "Canada Student Loans: How to Manage Your Debt

After Graduation" provides updates about federal government attempts to improve collections of student debt, and provides tips about responsible management of student loan repayment. "Beware of Payday Loans!" in Chapter 9 helps students understand that the convenience of so-called "payday loans" comes at a significant cost. Finally, in Chapter 13, "Mortgage Choices: Understanding the Options" introduces students to the decision making they will face when shopping for a residential mortgage, including fixed versus floating rates and shorter versus longer terms.

- **Updated rates and statistics:** This edition incorporates the most recent data available at the time of writing, including foreign exchange rates, Canada Savings bond rates, strip bond pricing, and current treatment of HST in Ontario, Prince Edward Island, New Brunswick, Nova Scotia, and Newfoundland and Labrador.

FEATURES

- **Check your Understanding feature:** Students are highly encouraged to maximize the utility of the Example problems throughout the text. At the end of each worked Example problem, a feature called "Check your Understanding" encourages students to *rework the question* using a slightly different set of parameters. The bottom-line answer is provided right in the feature, making it easy for students to check their work and accurately assess whether they understand the concept and are ready to move on. This feature is designed to help students become more *actively engaged* in the material as they make their way through the text.
- **Related problem:** Each worked Example problem directs students to a related problem in the end-of-section Exercise. Having read the worked example and reworked it using the "Check your Understanding" feature, students are then pointed to a problem in the Exercise that requires them to use the concept that they have just studied. This "read, do, and do again" approach will help students use the worked Example problems to support their learning.

EXAMPLE 2.1A Simplifying Algebraic Expressions by Combining Like Terms

a. $3a - 4b - 7a + 9b$	$3a$ and $-7a$ are like terms; $-4b$ and $9b$ are like terms.
$= 3a - 7a - 4b + 9b$	
$= (3 - 7)a + (-4 + 9)b$	Combine the numerical coefficients of like terms.
$= -4a + 5b$	
b. $0.2x + 5x^2 + \frac{x}{4} - x + 3$	Convert numerical coefficients to their decimal equivalents.
$= 5x^2 + (0.2 + 0.25 - 1)x + 3$	
$= 5x^2 - 0.55x + 3$	Then combine the like terms.
c. $\frac{2x}{1.25} - \frac{4}{5} - 1\frac{3}{4}x$	Convert numerical coefficients to their decimal equivalents.
$= 1.6x - 0.8 - 1.75x$	
$= -0.15x - 0.8$	Then combine the like terms.
d. $x\left(1 + 0.12 \times \frac{241}{365}\right) + \frac{2x}{1 + 0.12 \times \frac{81}{365}}$	Evaluate the numerical coefficients.
$= 1.07923x + \frac{2x}{1.02663}$	Combine the like terms.
$= (1.07923 + 1.94812)x$	
$= 3.02735x$	

Check your understanding: Simplify the following algebraic expression by combining like terms: $\frac{3x}{1.0164} + 1.049x - x$. Maintain five-figure accuracy. (Answer: $3.0006x$)

Related problem: #7 in Exercise 2.1

- **Categorized exercise problems:** Each section of the text provides a rich bank of practice problems for in-class or homework use. These problems are grouped into Basic, Intermediate, and Advanced sections. This will help instructors to present a "scaffolded" approach to each concept. Where time permits, faculty can use the advanced problems to illustrate extensions of the chapter concepts or, alternatively, to provide enrichment problems for students who seek extra challenge. A full set of end-of-chapter Review Problems is similarly categorized.

EXERCISE 2.3

Answers to the odd-numbered problems can be found in the end matter.

BASIC PROBLEMS

Solve the following equations.

1. $10a + 10 = 12 + 9a$ **2.** $29 - 4y = 2y - 7$

3. $0.5(x - 3) = 20$ **4.** $\frac{1}{3}(x - 2) = 4$

5. $y = 192 + 0.04y$ **6.** $x - 0.025x = 341.25$

7. $12x - 4(2x - 1) = 6(x + 1) - 3$ **8.** $3y - 4 = 3(y + 6) - 2(y + 3)$

9. $8 - 0.5(x + 3) = 0.25(x - 1)$ **10.** $5(2 - c) = 10(2c - 4) - 6(3c + 1)$

INTERMEDIATE PROBLEMS

Solve each of the following pairs of equations. Verify your solution in each case.

11. $x - y = 2$; $3x + 4y = 20$ **12.** $y - 3x = 11$; $5x + 30 = 4y$

13. $7p - 3q = 23$; $-2p - 3q = 5$ **14.** $y = 2x$; $7x - y = 35$

- **Math Apps:** These boxes, found in selected chapters, illustrate the application or misapplication of mathematics to business and personal finance.

Math Apps

WHAT TO DO WITH A SMALL AMOUNT OF SAVINGS IN A LOW INTEREST RATE ENVIRONMENT?

As we showed you in Chapter 6, interest rates in recent years have been at historically low levels when compared to the last 50 years. While this is good news for borrowers, low rates mean that savers are seeing dismal rates of return, especially if they keep their extra cash in traditional savings accounts. As a student, you are likely not faced with the enviable problem of trying to earn money on huge cash balances! Nevertheless, it is a good habit to start thinking about how to maximize every dollar you do have. Making smart choices about how to earn even a bit of extra money on small amounts of savings can help you develop a lifelong habit of smart investment choices.

Imagine that on September 1, you are starting the school year with a total of $10,000 in combined savings from your summer job and the proceeds of student loans. You plan to use half that money for your first semester of studies. The other half, $5000, will be saved to pay your second tuition installment that is due in early January. What should you do with the "extra" $5000 for the four months between September and early January? Since this money is needed for school, you do, of course, want an entirely safe investment. Investing in the stock market is not an option, as you know that, especially in the short run, stock prices can go up and down, and you do not want to risk losing any of this money. Your choices are therefore limited to short-term interest-based investments. After a bit of Internet research, you have found the following options:

- **Streamlined exposition:** In keeping with a "fundamentals" approach, explanations of chapter concepts have been streamlined to present the essence of each topic as succinctly as possible. For example, where more than one approach to a particular concept is available, students are made aware of both approaches but the "preferred" approach (as directed by reviewers) is the one that is expanded upon. This helps students to focus on the essential learning outcomes and avoid confusion that often results when multiple approaches are emphasized equally.

TECHNOLOGY

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- Access instructor resources.
- View assignments and resources created for past sections.
- Post your own resources for students to use.

INSTRUCTOR RESOURCES

- **Instructor's Solutions Manual:** Prepared by the author, with a technical review by Mariana Ionescu of George Brown College.
- **Computerized Test Bank:** Prepared by Julie Howse of St. Lawrence College.
- **Microsoft® PowerPoint® Lecture Slides:** Prepared by Sarah Chan.

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For more information, please visit us online at http://www.mheducation.ca/he/solutions.

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Business mathematics faculty across Canada are constantly adapting their course content and delivery to best serve their students. In this third edition of ***Fundamentals of Business Mathematics in Canada***, we have been assisted by many of those faculty who have shared their insights through thoughtful reviews and suggestions. We appreciate their ideas and applaud their tireless commitment to their students.

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Ernie Jerome & Jackie Shemko

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CHAPTER 1

Review and Applications of Basic Mathematics

LEARNING OBJECTIVES

After completing this chapter, you will be able to:

LO1 Perform arithmetic operations in their proper order

LO2 Convert fractions to their percent and decimal equivalents

LO3 Maintain the proper number of digits in calculations

LO4 Given any two of the three quantities percent rate, portion, and base, solve for the third

LO5 Calculate the gross earnings of employees paid a salary, an hourly wage, or commissions

LO6 Calculate the simple average or weighted average (as appropriate) of a set of values

LO7 Perform basic calculations for the Goods and Services Tax, Harmonized Sales Tax, provincial sales tax, and real property tax

connect

Throughout this chapter interactive charts and Help Me Solve It videos are marked with a ↗.

MOST ROUTINE CALCULATIONS IN BUSINESS are now performed electronically. Does this mean that mathematical skills are less important or less valued than in the past? Definitely not—the mathematics and statistics you study in your business program are more widely expected and more highly valued in business than ever before. Technology has empowered us to access more information more readily, and to perform more sophisticated analysis of the information. To take full advantage of technology, you must know which information is relevant, which analyses or calculations should be performed, how to interpret the results, and how to explain the outcome in terms that others can understand.

Employers and clients now expect higher education and performance standards from middle managers and advisors than in the past. Increasingly, employers also expect managers and management trainees to undertake a program of study leading to a credential recognized in their industry. These programs usually have a significant mathematics component.

Naturally, a college course in business mathematics or statistics will cover a broader range of topics (often in greater depth) than you might need for a particular industry. This broader education opens more career options to you and provides a stronger set of mathematical skills for your chosen career.

TIP

How to Succeed in Business Mathematics

In Connect, there are a guide and a video entitled "How to Succeed in Business Mathematics." These can be found in Library Resources under Course-wide Content.

1.1 Order of Operations

LO1 When evaluating an expression such as

$$5 + 3^2 \times 2 - 4 \div 6$$

there is potential for confusion about the sequence of mathematical steps. Do we just perform the indicated operations in a strict left-to-right sequence, or is some other order intended? To eliminate any possible confusion, mathematicians have agreed on the use of brackets and a set of rules for the order of mathematical operations. The rules are:

Rules for Order of Operations

1. Perform operations within brackets (in the order of Steps 2, 3, and 4 below).
2. Evaluate the powers.[1]
3. Perform multiplication and division in order from left to right.
4. Perform addition and subtraction in order from left to right.

[1] A power is a quantity such as 3^2 or 5^3 (which are shorthand methods for representing 3×3 and $5 \times 5 \times 5$, respectively). Section 2.2 includes a review of powers and exponents.

TIP

Using "BEDMAS"

To help remember the order of operations, you can use the acronym "BEDMAS" representing the sequence:

Brackets, **E**xponents, **D**ivision and **M**ultiplication, **A**ddition and **S**ubtraction.

TIP

Signs of Numbers

Pay careful attention to the signs associated with numbers. For example, in the expression 3 + (–4), the first term, 3, has a positive sign, while the term (–4) has a negative sign. We can simplify the expression by recognizing that adding a negative number is mathematically the same as subtracting that number. That is,

$$\begin{aligned} 3 + (-4) &= 3 - 4 \\ &= -1 \end{aligned}$$

When a positive number and a negative number are multiplied or divided, the result will be negative. For example, 3 × (–1) = –3. Similarly, –14 ÷ 2 = –7. However, when two negative numbers are multiplied or divided, the result will be positive. For example, (–3) × (–2) = 6 and (–18) ÷ (–9) = 2.

EXAMPLE 1.1A Exercises Illustrating the Order of Mathematical Operations

a. $\begin{aligned} 30 - 6 \div 3 + 5 &= 30 - 2 + 5 \\ &= 33 \end{aligned}$

Do division before subtraction and addition.

b. $\begin{aligned} (30 - 6) \div 3 + 5 &= 24 \div 3 + 5 \\ &= 8 + 5 \\ &= 13 \end{aligned}$

Do operations within brackets first; then do division before addition.

c. $\dfrac{30 - 6}{3 + 5} = \dfrac{24}{8} = 3$

Brackets are implied in the numerator and the denominator.

d. $\begin{aligned} 72 \div (3 \times 2) - 6 &= 72 \div 6 - 6 \\ &= 12 - 6 \\ &= 6 \end{aligned}$

Do operations within brackets first; then do division before subtraction.

e. $\begin{aligned} 72 \div (3 \times 2^2) - 6 &= 72 \div (3 \times 4) - 6 \\ &= 72 \div 12 - 6 \\ &= 6 - 6 \\ &= 0 \end{aligned}$

Do operations within brackets (the power before the multiplication); then do division before subtraction.

f. $\begin{aligned} 72 \div (3 \times 2)^2 - 6 &= 72 \div 6^2 - 6 \\ &= 72 \div 36 - 6 \\ &= 2 - 6 \\ &= -4 \end{aligned}$

Do operations within brackets first, then the power, then divide, then subtract.

g. $\left(\dfrac{32+8^2}{2\times 3}\right)-4=\left(\dfrac{32+64}{2\times 3}\right)-4$
$=\left(\dfrac{96}{6}\right)-4$
$=16-4$
$=12$

Do operations within brackets first, starting with the power. Brackets are implied in the numerator and the denominator. Divide, then subtract.

h. $3\times\left(\dfrac{21+43}{2\times 2}\right)^2=3\times\left(\dfrac{64}{4}\right)^2$
$=3\times 16^2$
$=3\times 256$
$=768$

Evaluate the numerator and denominator within the bracket. Then divide within the bracket. Evaluate the power. Then multiply.

Check your understanding: Evaluate the following: 4(2 – 5) – 4(5 – 2). (Answer: –24)

Related problem: #1 in Exercise 1.1

EXERCISE 1.1

Answers to the odd-numbered problems can be found in the end matter.

BASIC PROBLEMS

Evaluate each of the following. In Problems 24–28, evaluate the answers accurate to the cent.

1. $20-4\times 2-8$
2. $18\div 3+6\times 2$
3. $(20-4)\times 2-8$
4. $18\div(3+6)\times 2$
5. $20-(4\times 2-8)$
6. $(18\div 3+6)\times 2$
7. $54-36\div 4+2^2$
8. $(5+3)^2-3^2\div 9+3$
9. $(54-36)\div(4+2)^2$
10. $5+(3^2-3)^2\div(9+3)$
11. $\dfrac{8^2-4^2}{(4-2)^3}$
12. $1000(1+0.02)^3$
13. $(-1)(-3)+7-6^2$
14. $15(37-3^2)$

INTERMEDIATE PROBLEMS

15. $3(6+4)^2-5(17-20)^2$
16. $(4\times 3-2)^2\div(4-3\times 2^2)$
17. $[(20+8\times 5)-7\times(-3)]\div 9$
18. $5[19+(5^2-16)^2]^2$
19. $(9\times 3+32\div 2^2)^3$
20. $180-2[(-3\times 2^3+128\div 4^2)+6(-3+7)]$
21. $\dfrac{[8\times 6\div 4+(-3)\times 2]^2}{4+2}$
22. $\dfrac{3^4-7\times 2-6\times 4}{5}$
23. $7^2-\dfrac{13(-4)+7}{(-9)}$
24. $\$2000\left[\dfrac{(1+0.08)^4-(1+0.02)^4}{0.08-0.02}\right]$
25. $\$100\left(1+0.06\times\frac{45}{365}\right)$
26. $\dfrac{\$200}{1+0.09\times\frac{4}{12}}$

ADVANCED PROBLEMS

27. $\dfrac{\$600}{1+0.075\times\frac{250}{365}}$
28. $\$300\left[\dfrac{1-\frac{1}{(1+0.03)^2}}{0.03}\right]$
29. $\dfrac{28-2(3+2^2-2)}{6}+3\left(\dfrac{3\times 6^2}{4}\right)$
30. $\dfrac{4^2-2(-3+5)}{[(-6)\times 4+3(-1)+7]\div(-5)}$

1.2 Fractions

Definitions

In a fraction, the upper number and lower number are given the following labels:

$$\frac{\textbf{numerator}}{\textbf{denominator}}$$

Fractions can be labelled using the following categories:

Category	Examples	Description
Proper fraction	$\frac{3}{4}, \frac{1}{2}, \frac{3}{5}$	Numerator is smaller than denominator
Improper fraction	$\frac{6}{5}, \frac{3}{2}, \frac{11}{9}$	Numerator is larger than denominator
Mixed number	$2\frac{1}{3}, 8\frac{3}{5}, 13\frac{5}{8}$	A whole number plus a fraction
Equivalent fractions	$\frac{1}{2} = \frac{5}{10} = \frac{15}{30}$	Fractions that are equal in value, even though their respective numerators and denominators differ

EXAMPLE 1.2A Calculating an Equivalent Fraction

Find the missing numbers that make the following three fractions equivalent.

$$\frac{7}{12} = \frac{56}{?} = \frac{?}{300}$$

SOLUTION

To create a fraction equivalent to $\frac{7}{12}$, *both* the numerator and the denominator must be multiplied by the same number. To obtain 56 in the numerator of the second equivalent fraction, 7 was multiplied by 8. Hence, the denominator must also be multiplied by 8. Therefore,

$$\frac{7}{12} = \frac{7 \times 8}{12 \times 8} = \frac{56}{96}$$

To obtain the denominator (300) in the third equivalent fraction, 12 was multiplied by $\frac{300}{12} = 25$. The numerator must also be multiplied by 25. Hence, the equivalent fraction is

$$\frac{7 \times 25}{12 \times 25} = \frac{175}{300}$$

In summary,

$$\frac{7}{12} = \frac{56}{96} = \frac{175}{300}$$

Check your understanding: Find the missing number that makes the following fractions equivalent. $\frac{7}{12} = \frac{420}{?}$ (Answer: $\frac{7}{12} = \frac{420}{720}$)

Related problem: #1 in Exercise 1.2

LO2 Decimal and Percent Equivalents

To find the *decimal equivalent* value of a fraction, divide the numerator by the denominator. To express the fraction in *percent equivalent* form, shift the decimal point two places to the right (or multiply by 100) and add the % symbol.

EXAMPLE 1.2B Finding the Decimal and Percent Equivalents of Fractions and Mixed Number

Convert each of the following fractions and mixed numbers to its decimal equivalent and percent equivalent values.

a. $\frac{2}{5} = 0.4 = 40\%$

b. $\frac{5}{2} = 2.5 = 250\%$

c. $1\frac{3}{16} = 1.1875 = 118.75\%$

d. $\frac{3}{1500} = 0.002 = 0.2\%$

Check your understanding: Convert $3\frac{4}{5}$ to its decimal equivalent and percent equivalent values. (Answer: 3.8 and 380%)

Related problem: #4 in Exercise 1.2

TIP

Adding or Subtracting Fractions

To add or subtract fractions, the easiest approach is to first convert each fraction to its decimal equivalent value. Then add or subtract the decimal equivalents as required. For example, $\frac{2}{7} + \frac{252}{365} = 0.28571 + 0.69041 = 0.9761$ to four-figure accuracy.

LO3 Rounding of Decimal and Percent Equivalents

For some fractions, the decimal equivalent has an endless series of digits. Such a number is called a *nonterminating decimal*. In some cases, a nonterminating decimal contains a repeating digit or a repeating group of digits. This particular type of nonterminating decimal is called a *repeating decimal*. A shorthand notation for repeating decimals is to put a horizontal bar over the first occurrence of the repeating digit or group of digits. For example,

$$\frac{2}{9} = 0.222222 = 0.\overline{2} \quad \text{and} \quad 2\frac{4}{11} = 2.36363636 = 2.\overline{36}$$

When a nonterminating decimal or its percent equivalent is used in a calculation, the question arises: How many figures or digits should be retained? The following rules provide sufficient accuracy for the vast majority of our calculations.

Rules for Rounding Numbers

1. In intermediate results, keep at least one more figure than the number of figures required in the final result. (When counting figures for the purpose of rounding, do not count leading zeros[2] used only to properly position the decimal point.)
2. If the first digit dropped is 5 or greater, increase the last retained digit by 1.
3. If the first digit dropped is less than 5, leave the last retained digit unchanged.

[2] The following example illustrates the reasoning behind this instruction. A length of 6 mm is neither more nor less precise than a length of 0.006 m. (Recall that there are 1000 mm in 1 m.) The leading zeros in 0.006 m do not add precision to the measurement. They are inserted to properly position the decimal point. Both measurements have one-figure accuracy. Contrast this case with measurements of 1007 mm and 1.007 m. Here each zero comes from a decision about *what the digit should be (rather than where the decimal point should be). These measurements both have four-figure accuracy. This rule applies to the total number of figures (other than leading zeros) in a value. It does not apply to the number of decimal places.*

Suppose, for example, the answer to a calculation is expected to be a few hundred dollars and you want the answer accurate to the cent. In other words, you require five-figure accuracy in your answer. To achieve this accuracy, the first rule says you should retain (at least) six figures in values[3] used in the calculations. The rule also applies to intermediate results that you carry forward to subsequent calculations. The consequence of rounding can be stated in another way—if, for example, you use a number rounded to four figures in your calculations, you can expect only three-figure accuracy in your final answer.

TRAP

"Rounding Then Rounding"

Suppose you were asked to round 7.4999 to the nearest whole number. Avoid the trap of "rounding then rounding." For example, some students will begin by rounding 7.4999 to the nearest tenth, to give 7.5. Then they will round that *already rounded* number to the nearest whole, which is 8. This approach is incorrect. When asked to round any number, look only at the first digit beyond the required degree of accuracy. Rounding 7.4999 to the nearest whole number, we should look only at the *first* digit after the decimal sign. Since the first digit being dropped (in this case, the first digit after the decimal) is less than 5, leave the last retained digit unchanged. Some people refer to this as "rounding down." Therefore, 7.4999 rounded to the nearest whole is 7, not 8.

EXAMPLE 1.2C Fractions Having Repeating Decimal Equivalents

Convert each of the following fractions to its decimal equivalent value expressed in the repeating decimal notation.

a. $\frac{14}{9} = 1.555\cdots = 1.\overline{5}$

b. $3\frac{2}{11} = 3.181818\cdots = 3.\overline{18}$

c. $5\frac{2}{27} = 5.074074\cdots = 5.\overline{074}$

d. $\frac{5}{7} = 0.714285714285\cdots = 0.\overline{714285}$

Check your understanding: Convert the fraction $\frac{11}{12}$ to its decimal equivalent form in the repeating decimal notation. (Answer: $0.91\overline{6}$)

Related problem: #13 in Exercise 1.2

EXAMPLE 1.2D Calculating and Rounding the Decimal Equivalents of Fractions

Convert each of the following fractions and mixed numbers to its decimal equivalent value rounded to four-figure accuracy.

a. $\frac{2}{3} = 0.6667$

b. $6\frac{1}{12} = 6.083$

c. $\frac{173}{11} = 15.73$

d. $\frac{2}{1071} = 0.001867$

e. $\frac{17{,}816}{3} = 5939$

Check your understanding: Convert the fraction $\frac{3}{365}$ to its decimal equivalent value rounded to *five-figure* accuracy. (Answer: 0.0082192)

Related problem: #25 in Exercise 1.2

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[3] Some values may be known and written with perfect accuracy in less than five figures. For example, a year has exactly 12 months, or an interest rate may be exactly 6%.

EXAMPLE 1.2E Demonstrating the Consequences of Too Much Rounding

Accurate to the cent, evaluate

$$\$140\left(1 + 0.11 \times \tfrac{113}{365}\right) + \$74\left(1 + 0.09 \times \tfrac{276}{365}\right)$$

SOLUTION

If we first evaluate the contents of the brackets, we obtain

$$\$140(1.0340548) + \$74(1.0680548)$$

With just a crude estimation, we can say that the first product will be a little greater than $140 and the second product will be a little greater than $74. Therefore, the answer will be between $200 and $300. If you want the answer to be accurate to the cent, then you are seeking *five*-figure accuracy. If you wish to round the values used, Rule 1 says that you must keep at least *six* figures in intermediate values. That is, you should round no further than

$$\begin{aligned}\$140(1.03405) + \$74(1.06805) &= \$144.767 + \$79.0357\\ &= \$223.8027\\ &= \$223.80 \text{ (rounded to the cent)}\end{aligned}$$

Suppose instead that you round to three figures, perhaps thinking that keeping two decimal places will give you an answer correct to two decimal places (that is, accurate to the cent.) You will obtain

$$\begin{aligned}\$140(1.03) + \$74(1.07) &= \$144.20 + \$79.18\\ &= \$223.38\end{aligned}$$

This answer is in error by $0.42.

What if you keep five figures (in the hope that the answer will be as accurate as the values used to calculate it)? Your result will be

$$\begin{aligned}\$140(1.0341) + \$74(1.0681) &= \$144.774 + \$79.0394\\ &= \$223.8134\\ &= \$223.81 \text{ (rounded to the cent)}\end{aligned}$$

This answer is in error by $0.01. The trend we observe here is typical—the more figures you keep in intermediate steps of a calculation, the smaller the error in your final answer.

There is one more point worth noting. Consider the first line in the initial calculation where you properly maintained six-figure accuracy. That is,

$$\$140(1.03405) + \$74(1.06805) = \$144.767 + \$79.0357$$

Suppose you round the two amounts on the right side to the nearest cent *before* you add them. The sum is then

$$\$144.77 + \$79.04 = \$223.81$$

which is $0.01 larger than the correct answer. The error arises because, just at the final addition, you failed to maintain six-figure accuracy (to ensure five-figure accuracy in the final answer).

In summary, if you want five-figure accuracy in your answer, you cannot round to less than six figures *at any stage* of the calculations. (You can verify that keeping more than six figures will not change the answer at the fifth figure.)

Check your understanding: Accurate to the cent, evaluate $\$1388\left(1 + 0.05 \times \tfrac{112}{365}\right) + \$50\left(1 + 0.04 \times \tfrac{90}{365}\right)$. (Answer: $1459.79)

Related problem: #33 in Exercise 1.2

TIP

Optimal Use of Your Calculator

Whenever possible, use your calculator's memory registers to save intermediate results. This will save time and reduce keystroke errors during data re-entry. It also virtually eliminates the introduction of rounding errors, since most calculators internally retain two or three more figures than are shown in the display. Example 1.2F illustrates this approach.

EXAMPLE 1.2F Optimal Use of Your Calculator

We will again evaluate (accurate to the cent) the same expression as in Example 1.2E,

$$\$140\left(1 + 0.11 \times \tfrac{113}{365}\right) + \$74\left(1 + 0.09 \times \tfrac{276}{365}\right)$$

This time we will use our financial calculator in a way that (1) avoids manual re-entry of intermediate results and (2) maintains maximum precision by avoiding rounding (other than rounding imposed by the inherent limitation of the calculator).

SOLUTION

We assume the Texas Instruments BA II PLUS calculator is set for a floating-decimal format and for the algebraic operating system (AOS) calculation method. (Refer to Appendix 1A for instructions on making these settings.) In the AOS mode, we can enter numbers, brackets, and operations in the same left-to-right sequence as they are written in. The calculator performs the calculations according to the proper order of operations.

140 [×] [(] 1 [+] 0.11 [×] 113 [÷] 365 [)]

[+] 74 [×] [(] 1 [+] 0.09 [×] 276 [÷] 365 [)] [=] 223.80

The result is \$223.80.

You see that it is possible to evaluate quite complex expressions without writing down intermediate results. However, if someone is going to read and readily understand your solution, you should present enough detail and intermediate results to reveal the steps in your solution.

Check your understanding: Accurate to the cent, evaluate $\$398\left(1 + 0.09 \times \tfrac{5}{12}\right) + \$50\left(1 + 0.04 \times \tfrac{11}{12}\right)$. (Answer: \$464.76)

Related problem: #35 in Exercise 1.2

Evaluating Complex Fractions

A **complex fraction** is a fraction containing one or more other fractions in its numerator or denominator. In simplifying complex fractions, pay particular attention to the correct order of mathematical operations, as discussed in Section 1.1.

EXAMPLE 1.2G Evaluating Complex Fractions

Evaluate each of the following complex fractions accurate to the cent.

a. $\dfrac{\$425}{\left(1+\frac{0.09}{12}\right)^{24}}$ **b.** $\dfrac{\$1265\left(1+0.115\times\frac{87}{365}\right)}{1+0.125\times\frac{43}{365}}$

SOLUTION

We assume the Texas Instruments BA II PLUS calculator is set for a floating-decimal format and for the algebraic operating system (AOS) calculation method. (Refer to Appendix 1A for instructions on making these settings.)

a. 425 [÷] [(] 1 [+] 0.09 [÷] 12 [)] [y^x] 24 [=] 355.23

The result is \$355.23.

b. 1265 [×] [(] 1 [+] 0.115 [×] 87 [÷] 365 [)]

[÷] [(] 1 [+] 0.125 [×] 43 [÷] 365 [)] [=] 1280.81

The result is \$1280.81.

Check your understanding: Evaluate $\dfrac{\$200\left(1+0.07\times\frac{30}{365}\right)}{1+0.085\times\frac{225}{365}}$ with accuracy to the nearest cent. (Answer: \$191.14)

Related problem: #39 in Exercise 1.2

Checkpoint Questions

1. Circle "True" or "False" for each of the following:
   - **a.** The number 0.00312 has five-figure accuracy. True False
   - **b.** The number 1.000047 has seven-figure accuracy. True False
   - **c.** The number 100.38 has two-figure accuracy. True False
   - **d.** The fraction $\frac{12}{49}$ is equivalent to the fraction $\frac{156}{637}$. True False
   - **e.** The fraction $\frac{6}{16}$ is equivalent to the fraction $\frac{126}{240}$. True False
   - **f.** The fraction $\frac{8}{3}$ is a proper fraction. True False
   - **g.** The value $2\frac{1}{3}$ is a mixed number. True False
2. If you want four-figure accuracy in your final result, what minimum number of figures must be retained in the values used in the calculations?
3. For a final result of approximately \$7000 to be accurate to the cent, what minimum number of figures must be retained in the values used in the calculations?
4. If a final result on the order of five million dollars is to be accurate to the nearest dollar, what minimum number of figures must be retained in the calculations?
5. If an interest rate (which might be greater than 10%) is to be calculated to the nearest 0.01%, what minimum number of digits must be retained in the numbers used to calculate the interest rate?

EXERCISE 1.2

Answers to the odd-numbered problems can be found in the end matter.

BASIC PROBLEMS

Fill in the missing numbers to create sets of equivalent fractions.

1. $\frac{3}{8} = \frac{12}{?} = \frac{?}{120}$ **2.** $\frac{9}{13} = \frac{54}{?} = \frac{?}{143}$ **3.** $\frac{8}{9} = \frac{?}{279} = \frac{488}{?}$

The following fractions and mixed numbers have *terminating* decimal equivalent forms. Express their decimal and percent equivalent forms to five-figure accuracy.

4. $\frac{7}{8}$ **5.** $\frac{47}{20}$ **6.** $-\frac{9}{16}$ **7.** $\frac{-35}{25}$

8. $1\frac{7}{25}$ **9.** $\frac{25}{1000}$ **10.** $-1\frac{11}{32}$ **11.** $12\frac{5}{8}$

The following fractions and mixed numbers have *repeating* decimal equivalent forms. Express their decimal and percent equivalent forms in the repeating decimal notation. Show just the minimum number of decimal places needed to display the repeating digit or group of digits.

12. $\frac{5}{6}$ **13.** $-\frac{8}{3}$ **14.** $1\frac{1}{11}$ **15.** $\frac{37}{27}$

INTERMEDIATE PROBLEMS

Round each of the following to four-figure accuracy.

16. 11.3845 **17.** 9.6455 **18.** 0.5545454 **19.** 1000.49

20. 1.0023456 **21.** 0.030405 **22.** 40.09515 **23.** 0.0090909

Convert each of the following fractions and mixed numbers to its decimal equivalent and percent equivalent values, rounded to five-figure accuracy.

24. $\frac{7}{6}$ **25.** $\frac{1}{60}$ **26.** $\frac{15}{365}$ **27.** $\frac{0.095}{12}$

28. $3\frac{12}{19}$ **29.** $6\frac{1}{17}$ **30.** $\frac{3}{7}$ **31.** $1\frac{0.035}{12}$

Evaluate each of the following, accurate to the nearest cent.

32. $\$92(1 + 0.095 \times \frac{112}{365})$

33. $\$100(1 + 0.11 \times \frac{5}{12}) + \$87(1 + 0.08 \times \frac{7}{12})$

34. $\$454.76(1 - 0.105 \times \frac{11}{12})$

35. $\frac{\$3490}{1 + 0.125 \times \frac{91}{365}}$

36. $\frac{\$10{,}000}{1 - 0.10 \times \frac{182}{365}}$

37. $\$650\left(1 + \frac{0.105}{2}\right)^2$

38. $\frac{\$15{,}400}{(1 + \frac{0.13}{12})^6}$

39. $\frac{\$550}{(1 + \frac{0.115}{2})^4}$

ADVANCED PROBLEMS

40. $\frac{\$6600(1 + 0.085 \times \frac{153}{365})}{1 + 0.125 \times \frac{82}{365}}$

41. $\$1000\left[\frac{(1 + \frac{0.09}{12})^7 - 1}{\frac{0.09}{12}}\right]$

42. $\$475\left[\frac{(1 + \frac{0.075}{12})^{2\frac{1}{2}} - 1}{\frac{0.075}{12}}\right]$

43. $\frac{\$17{,}500(1 + 0.0475 \times 2\frac{187}{365})}{1 + 0.0875 \times \frac{197}{365}}$

1.3 The Basic Percentage Problem

LO4 Often we wish to compare a portion, or part of a quantity, to the whole amount. One measure of the relative size is the fraction

$$\frac{\text{Portion}}{\text{Base}}$$

where the term *Base* is used to represent the whole or entire amount. The fraction is called the *Rate*. That is,

THE BASIC PERCENTAGE FORMULA

$$\textbf{Rate} = \frac{\textbf{Portion}}{\textbf{Base}} \qquad \textbf{(1-1)}$$

This relation is also used in a more general way to compare a quantity (the *Portion*) to some other standard or benchmark (the *Base*). In these cases the *Portion* may be larger than the *Base*. Then the *Rate* will be greater than 1 and the percent equivalent *Rate* will be more than 100%.

TRAP

Decimal Equivalent of Rates Smaller Than 1%

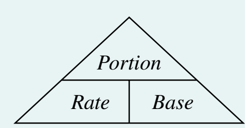
When a *Rate* is less than 1%, students sometimes forget to move the decimal two places to the left in order to obtain the decimal equivalent *Rate*. For example, be clear on the distinction between 0.25% and 25%. The former is just $\frac{1}{4}$ of 1%—the latter is 25 *times* 1%. Their decimal equivalents are 0.0025 and 0.25, respectively.

Given any two of the three quantities: *Portion*, *Base*, and *Rate*, you can calculate the unknown quantity by using formula (1-1). *First* substitute the known values, and *then* rearrange the equation to solve for the unknown.

TIP

The Portion, Rate, Base Triangle

In the examples in this section, we will substitute known values into formula (1-1) and rearrange to solve for the unknown. It is important to become comfortable with rearranging formulas, and we encourage you to practise this important basic skill. Then, you may find it saves time to use the diagram shown here when solving problems involving rate, portion, and base.



When you have determined which variable you are being asked to find, cover up that variable in the triangle. The remaining variables will appear in the correct order to help you solve for the unknown value you are seeking. For example, if a question requires you to solve for *Portion*, covering the word *Portion* in the triangle leaves the words *Rate* and *Base* side by side, which indicates those two values must be multiplied. If another question requires you to solve for *Base*, covering up the word *Base* in the triangle leaves *Portion* above the word *Rate*, which indicates that you must use the fraction $\frac{\text{Portion}}{\text{Rate}}$ to solve for *Base*. Finally, if asked to solve for *Rate*, covering that word in the triangle reveals the word *Portion* above the word *Base*, which indicates you must use the fraction $\frac{\text{Portion}}{\text{Base}}$ to solve for *Rate*.

TIP

Using the Word "Of" to Distinguish Between the Base and the Portion

The key to solving percentage problems is to distinguish between the *Base* and the *Portion*. The *Base* is always the standard or benchmark to which the *Portion* is being compared. In the wording of problems, the quantity following the word "of" is almost always the *Base*. For example, consider the following questions:

"What is 12% of \$993?"	The value after the word "of" is \$993, which is the *Base*. We are being asked to solve for the *Portion*.
"75 is 8% of what number?"	The words "what number" come after the word "of," so we are being asked to solve for the *Base*. The value 75 represents the *Portion*.

EXAMPLE 1.3A Using the Basic Percentage Formula

a. What is $40\frac{1}{4}\%$ of \$140.25?

b. How much is $0.08\overline{3}\%$ of \$5000?

c. What percentage is 7.38 kg of 4.39 kg?

SOLUTION

a. The question asks us to calculate a part (*Portion*) of a given whole (*Base*). The value \$140.25 appears after the word "of," and is the base—the benchmark to which the unknown value, *Portion*, is being compared. The rate is $40\frac{1}{4}\%$, which, when written in its decimal equivalent form, is 0.4025. Substituting the known values into formula (1-1):

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}}$$

we obtain

$$0.4025 = \frac{\text{Portion}}{\$140.25}$$

Multiply both sides by \$140.25 to get rid of the fraction on the right side of the equation:

$$0.4025 \times \$140.25 = \frac{\text{Portion}}{\$140.25} \times \$140.25$$

$$\$56.4506 = \text{Portion}$$

Switching the left and right sides, and then rounding to the cent, we get Portion = \$56.45.

That is, $40\frac{1}{4}\%$ of \$140.25 is \$56.45.

b. This question asks us to calculate a part (Portion) given the whole (Base) and rate. As in Part (a), we must substitute the known values into formula (1-1):

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}}$$

Note that when writing a rate or percentage in a formula, we must first convert it to its decimal equivalent. Divide the percentage, $0.08\overline{3}\%$, by 100 to arrive at its decimal equivalent, $0.0008\overline{3}$. Substituting into formula (1-1) we obtain

$$0.0008\overline{3} = \frac{\text{Portion}}{\$5000}$$

$$Portion = 0.0008\overline{3} \times \$5000 = 0.0008333 \times \$5000 = \$4.17$$

Here is the line of self-questioning and thinking behind rounding the repeating decimal at the fourth figure. (Remember not to count the three leading zeros in 0.0008333, because they serve only to position the decimal point.)

Question, Q: What accuracy do we want in the answer?

Answer, A: We want the answer to be accurate to the cent. (This is the normal understanding in financial calculations.)

Q: How many digits or figures of accuracy does the preceding answer imply?

A: This number of digits depends on the size of the answer. So we must first estimate that size. 0.083% is a little less than 0.1% (one-tenth of one percent). Since 1% of $5000 is $50, then 0.1% of $5000 is only $5. Therefore, the answer will be a little less than $5. For the answer to be accurate to the cent, we seek *three-figure* accuracy.

Q: How many figures of accuracy must we maintain throughout the calculations?

A: The fundamental rule is to keep at least one more figure of accuracy than is required in the final answer. Therefore, we must maintain at least four-figure accuracy in the calculations.

In conclusion, $4.17 is $0.08\overline{3}$% of $5000.

c. We are given both the *Portion* and the *Base* for a comparison. Here 7.38 kg is being compared to the reference amount (Base) of 4.39 kg. The answer will be greater than 100%, since the Portion is larger than the Base.

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}} = \frac{7.38}{4.39} = 1.681 = 168.1\%$$

Thus, 7.38 kg is 168.1% of 4.39 kg.

Check your understanding: 250% of what amount is $10? (Answer: $4)

Related problem: #3 in Exercise 1.3

EXAMPLE 1.3B A Word Problem Requiring the Basic Percentage Formula

A battery manufacturer encloses a 50-cent rebate coupon in a package of two AAA batteries retailing for $4.29. What percent rebate does the coupon represent?

SOLUTION

In effect, the question is asking you to compare the rebate to the retail price. Therefore, the retail price is the *Base* in the comparison.

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}} = \frac{\$0.50}{\$4.29} = 0.117 = 11.7\%$$

The manufacturer's percent rebate on the batteries is 11.7%.

Check your understanding: If a 75-cent rebate coupon represents a savings of 4% of the regular retail price, what was the retail price of the item before the rebate? (Answer: $18.75)

Related problem: #23 in Exercise 1.3